Warm-Up

CST/CAHSEE: Gr. 6 AF 1.4	Review: Gr. 7 NS 1.2
Simplify: $8 + 8 \div 2 + 2$ A) 4 B) 8 C) 10 D) 14 How did students get the other answers?	Complete the statement using <, >, =. Explain. $2^5 \bigcirc 5^2$
Other: Gr. 7 NS 1.2	**Current: Gr. 6 AF 1.4**
Evaluate when $x = -3$. Use multiple representations. $7 - (x)$	Simplify the expression. $8 - 2(3)$

Today's Objective/Standards: Students will use the correct order of operations to evaluate algebraic expressions/ Gr. 6 AF 1.4

Order of Operations

I. **Warm-Up:**

After warm-up, use an overhead basic calculator and punch in CST problem. Then have a student punch in the same problem into a scientific calculator. Are the answers the same? Why not?

II. **What is "Order of Operations?"**

Teacher will ask students, "What is Order of Operations?' Students will share their thoughts with their partner and then report out. **Order of Operations** is the sequence in which operations in a mathematical expression are evaluated or simplified. Simplifying a mathematical expression without order of operations would be like playing football or baseball without rules. Without **rules of the game**, there would be no structure or order.

We use the acronym **GEMDAS** to help students remember "Order of Operations":

G	Evaluate grouping symbols	$(\ ), [\ ], \{\ \}, \lvert\ \rvert, \text{——}$
E	Evaluate powers	$x^2, \sqrt{\ }$
$\overrightarrow{\text{MD}}$	Multiplication or Division, whichever comes first, left to right	$\bullet, \times, (\ )(\ ), 3(x+2), \div, -$
$\overrightarrow{\text{AS}}$	Addition or Subtraction, whichever comes first, left to right	$+, -$

- Addition is the simplest operation, and therefore, the lowest level in our order of operations. Subtraction is just adding the opposite.

$$\begin{aligned} & 6-7 \\ &= 6+(-7) \\ &= -1 \end{aligned}$$

Multiplication is repeated addition.

$$\begin{aligned} & 3 \cdot 2 \\ &= 2 + 2 + 2 \\ &= 6 \end{aligned}$$

- Exponents are repeated multiplication (or repeated division in the case of negative exponents)- so again, another level of complexity.

$$\begin{aligned} & 2^3 \\ &= 2 \cdot 2 \cdot 2 \\ &= 8 \end{aligned}$$

- Grouping Symbols contain a quantity, and that quantity must be considered as one, although it could have a number of terms.

- Distribution really allows us to "change" the order of operations from working within the parenthesis first to multiplying to each term, so as to see the quantity in parts, but equivalent to the one quantity.

$$\begin{aligned} & 2(x+5) \\ &= 2(x) + 2(5) \\ &= 2x + 10 \end{aligned}$$

OR

$$\begin{aligned} & 2(x+5) \\ &= (x+5) + (x+5) \\ &= x + x + 5 + 5 \\ &= 2(x) + 2(5) \\ &= 2x + 10 \end{aligned}$$

III. **Using Order of Operations to Simplify Expressions:**

Ex. 1)

$$\begin{aligned} & 16 - 20 \div 2^2 \\ &= 16 - 20 \div 4 \\ &= 16 - 5 \\ &= 11 \end{aligned}$$

G

~~E~~

M ~~D~~

A ~~S~~

Circle the corresponding letter as students respond. Cross off the letter as you perform.

Standard Questions: Choral Response

"Do we have grouping?" NO

"Do we have exponents?" YES

"Do we have multiplication and/or division?" YES

"Do we have addition and/ or subtraction?" YES

Ex. 2)

$$
\begin{aligned}
& 38 - 2(7 - 5 + 3) \\
&= 38 - 2(2 + 3) \\
&= 38 - 2(5) \\
&= 38 - 10 \\
&= 28
\end{aligned}
$$

~~G~~ → G

E   E

~~M~~ D   M D

A ~~S~~   ~~A~~ ~~S~~

"Do we have grouping?" YES

Follow above standard questions

Note: If a grouping symbol appears, you can do the GEMDAS analysis within the grouping symbol(s).

Ex. 3)

$$
\begin{aligned}
& \frac{24 \bullet 3}{5 + 3^2 - 2} \\
&= \frac{72}{5 + 3^2 - 2} \\
&= \frac{72}{5 + 9 - 2} \\
&= \frac{72}{14 - 2} \\
&= \frac{72}{12} \\
&= 6
\end{aligned}
$$

Note: the fraction bar acts as a grouping symbol for both numerator and denominator.

Follow above standard questions for each example. Go to "You Try" when you feel your students are ready.

Ex. 4)

$$
\begin{aligned}
& \left[\left(4^2 + 2\right) \div 2 + 10\right] - 2 \\
&= \left[(16 + 2) \div 2 + 10\right] - 2 \\
&= \left[18 \div 2 + 10\right] - 2 \\
&= \left[9 + 10\right] - 2 \\
&= 19 - 2 \\
&= 17
\end{aligned}
$$

Focus on Proper Syntax.

YOU TRIES: Students will work in pairs and be asked to simplify the expressions below. One student will be the recorder and the other student will tell the recorder what to write down. For the second expression, students will switch roles. Both will simplify the third "You Try." This will lower affective filter.

1) $24 + 4(3 + 1)$ 2) $\frac{3}{8}\left[13 - (2 + 3)\right]^2$ 3) $8\left[20 - (9 - 5)^2\right]$

IV. **Mitigation**: Common mistakes can be avoided if we address them to help develop student understanding.

Below are just a few examples. We must continue to emphasize **Order of Operations** when simplifying any expression. This will help students avoid mistakes as they move onto Algebra I.

Ex. 1)

$$\frac{7-2\bullet 3}{7+3\bullet 4}$$
$$=\frac{7-6}{7+3\bullet 4}$$
$$=\frac{1}{7+3\bullet 4}$$
$$=\frac{1}{7+12}$$
$$=\frac{1}{19}$$

Error analysis:

$$\frac{\cancel{7}-2\bullet 3}{\cancel{7}+3\bullet 4}$$
$$=\frac{-2\bullet 3}{3\bullet 4}$$
$$=\frac{-1\bullet 2\bullet 3}{2\bullet 2\bullet 3}$$
$$=-\frac{1}{2}$$

Is this correct? No
What is wrong? The 7s are part of quantities and are not equivalent forms of one.

Students may see the $\frac{7}{7}$ as an equivalent form of one. For that reason teachers will group the numerator and denominator to show that each is one quantity. Remind students that only factors/factors can be simplified as equivalent forms of one, not terms.

$$\frac{\cancel{2}\bullet 3}{\cancel{2}\bullet 5}$$
$$=\frac{3}{5}$$

Example 1 brings up another common problem: $\frac{x+2}{2}$. Many students will do the following:

$$\frac{x+2}{2}$$
$$=\frac{x+\cancel{2}}{\cancel{2}}$$
$$=x$$

Note:

$$\frac{x+2}{2}=\frac{x}{2}+\frac{2}{2}$$
$$=\frac{x}{2}+1$$
$$\neq x$$

$x+2$ must be seen as one quantity. You can decompose the expression $\frac{x+2}{2}$.

$$\frac{x+2}{2}$$
$$=\frac{x}{2}+\frac{2}{2}$$
$$=\frac{x}{2}+1$$

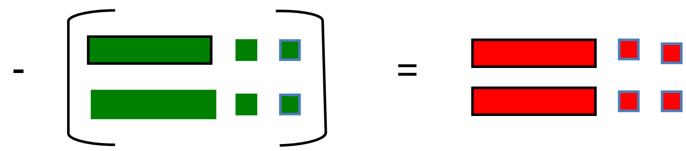
Ex. 2)

$$-2(x+2)$$
$$= -2(x) + -2(2)$$
$$= -2x + (-4)$$
$$= -2x - 4$$

Students need to see $-2(x+2)$ as multiplication. Because we emphasize simplifying expressions in grouping symbols, students will make mistakes in regards to combining like terms. Students may call $x+2$, $2x$. Tell students that we have to use distributive property because we cannot combine unlike terms, in this case, x and 2.

The opposite of 2 groups of $(x+2)$

$$= -[(x+2)+(x+2)]$$
$$= -[2x+4]$$
$$= -2x - 4$$



Ex. 2 brings up another issue: If we have $5-2(x+2)$, some students will subtract first instead of multiplying using the distributive property.

Wrong	Correct
$5-2(x+2)$	$5-2(x+2)$
$= 3(x+2)$	$= 5-2x-4$
$= 3x+6$	$= -2x+1$

What are we starting with? 5

What are we subtracting? 2 groups of ($x+2$)

Ex. 3)

The opposite of 3^2

$$-3^2$$
$$= -(3)(3)$$
$$= -9$$

Versus

$$(-3)^2$$
$$= (-3)(-3)$$
$$= 9$$

In the expression -3^2, 3 is the base. In the expression $(-3)^2$, -3 is the base.

Ex. 4)

Wrong	Correct
$7-(-4)^2$	$7-(-4)^2$
$= 7+(4)^2$	$= 7-(-4)(-4)$
$= 11^2$	$= 7-16$
$= 121$	$= -9$

What are we starting with? 7

What are we subtracting? $(-4)^2$

V. Inserting Parenthesis

$21 - 10 \bullet 2 = 22$

"Is this correct?" No

"How did they get their answer?"
They subtracted first.

"Can you insert a grouping symbol to make the equation true?"
$(21 - 10) \bullet 2 = 22$

You Tries: Insert grouping symbols to make the equation true.

4) $3^3 - 15 \div 5 + 2 = 22$

5) $48 \div 2 \bullet 12 - 4 = 3$

EXTRA PROBLEMS:

1. $\frac{21+15}{3+6}$

2. $2\left[8 + (5 - 3)\right] - 8$

3. $15 + (4 + 6)^2 \div 5$

4. $(4 + 8)^2 \div 4^2$

5. Evaluate when $y = -6$.
   $y^2 + 2y + 5$

6. $24 + (11 - 3)^2 \div 4$

7. $(5 + 10)^2 \div 5^2$

8. $9 \bullet 5 - 4(12 \div 6)$

9. $3 \bullet 7 + 6 \div 2$

10. $\frac{60 \div 4 + 9}{12 \div 3 - 2 + 1}$

Answers to "You Tries"

1.
$$
\begin{aligned}
&24+4(3+1)\\
&=24+4(4)\\
&=24+16\\
&=40
\end{aligned}
$$

2.
$$
\begin{aligned}
&\frac{3}{8}\left[13-(2+3)\right]^2\\
&=\frac{3}{8}\left[13-5\right]^2\\
&=\frac{3}{8}\left[8\right]^2\\
&=\frac{3}{8}\left[64\right]\\
&=\frac{3\bullet 8\bullet 8}{8}\\
&=24
\end{aligned}
$$

3.
$$
\begin{aligned}
&8\left[20-(9-5)^2\right]\\
&=8\left[20-(4)^2\right]\\
&=8\left[20-16\right]\\
&=8\left[4\right]\\
&=32
\end{aligned}
$$

4.
$$
\begin{aligned}
3^3-(15\div 5+2)&=22\\
3^3-(3+2)&=22\\
3^3-5&=22\\
27-5&=22\\
22&=22
\end{aligned}
$$

5.
$$
\begin{aligned}
48\div\left[2\bullet(12-4)\right]&=3\\
48\div\left[2\bullet(8)\right]&=3\\
48\div 16&=3\\
3&=3
\end{aligned}
$$

Answers to Extra Problems:

1. $$\frac{21+15}{3+6}$$
$$=\frac{36}{3+6}$$
$$=\frac{36}{9}$$
$$=4$$

2. $$2\left[8+(5-3)\right]-8$$
$$=2\left[8+2\right]-8$$
$$=2\left[10\right]-8$$
$$=20-8$$
$$=12$$

3. $$15+(4+6)^2\div 5$$
$$=15+(10)^2\div 5$$
$$=15+100\div 5$$
$$=15+20$$
$$=35$$

4. $$(4+8)^2\div 4^2$$
$$=(12)^2\div 4^2$$
$$=144\div 4^2$$
$$=144\div 16$$
$$=9$$

5. Evaluate when $y=-6$
$$y^2+2y+5$$
$$=(-6)^2+2(-6)+5$$
$$=36+2(-6)+5$$
$$=36-12+5$$
$$=24+5$$
$$=29$$

6. $$24+(11-3)^2\div 4$$
$$=24+(8)^2\div 4$$
$$=24+64\div 4$$
$$=24+16$$
$$=40$$

7. $$(5+10)^2\div 5^2$$
$$=(15)^2\div 5^2$$
$$=225\div 5^2$$
$$=225\div 25$$
$$=9$$

8. $$9\bullet 5-4(12\div 6)$$
$$=9\bullet 5-4(2)$$
$$=45-4(2)$$
$$=45-8$$
$$=37$$

9. $$3\bullet 7+6\div 2$$
$$=21+6\div 2$$
$$=21+3$$
$$=24$$

10. $$\frac{60\div 4+9}{12\div 3-2+1}$$
$$=\frac{15+9}{12\div 3-2+1}$$
$$=\frac{24}{4-2+1}$$
$$=\frac{24}{2+1}$$
$$=\frac{24}{3}$$
$$=8$$

Answers to Warm-Up

Other: Evaluate when $x = -3$

$$\begin{aligned} & 7-(x) \\ &= 7-(-3) \\ &= 7+3 \\ &= 10 \end{aligned}$$

Current:

$$\begin{aligned} & 8-2(3) \\ &= 8-6 \\ &= 2 \end{aligned}$$

Review: $2^5 > 5^2$

$$\begin{aligned} & 2^5 \\ &= (2)(2)(2)(2)(2) \\ &= 32 \end{aligned}$$

$$\begin{aligned} & 5^2 \\ &= (5)(5) \\ &= 25 \end{aligned}$$

CST/CAHSEE: D

$$\begin{aligned} & 8+8 \div 2+2 \\ &= 8+4+2 \\ &= 12+2 \\ &= 14 \end{aligned}$$